# Structure-Preserving Signatures on Equivalence Classes From Standard Assumptions

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ASIACRYPT 2019, Kobe, Japan, December 11, 2019







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- Structure-Preserving Signatures and Applications
- Structure-Preserving Signatures on Equivalence Classes
- Overview of the State-of-the-Art
- Our Approach
- Take Home & Open Questions

# Structure-Preserving Signatures and Applications

#### Bilinear groups

 $\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T$  are cyclic groups of prime order p

 $\boldsymbol{\cdot} \ \boldsymbol{e}: \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ 

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 $\sigma \leftarrow \mathsf{Sign}(\mathsf{sk},m) : m \in \mathbb{G}_i^n; \sigma \in \mathbb{G}_1^u \times \mathbb{G}_2^v$   
 $\{\mathsf{0},\mathsf{1}\} \leftarrow \mathsf{Verify}(\mathsf{pk},m,\sigma) : \mathsf{Only uses}$ 

pairing-product equations

$$\prod_i \prod_j e(A_i, \hat{B}_j)^{a_{ij}} = Z$$
, and

group membership tests.

#### Compatible with efficient Groth-Sahai (GS) NIZK proofs

#### Numerous privacy-preserving applications

(Delegatable) anonymous credentials, group signatures, traceable signatures, blind signatures, anonymous e-cash, verifiable shuffles (e-voting), etc.



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The equivalence relation  $\sim_{\mathcal{R}}$ 

$$\mathbf{m} \in (\mathbb{G}_i^*)^\ell \sim_{\mathcal{R}} \mathbf{n} \in (\mathbb{G}_i^*)^\ell \Leftrightarrow \exists \mu \in \mathbb{Z}_p^* : \mathbf{m} = \mu \mathbf{n}$$

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### Unlinkability on message space

• No advantage in distinguishing classes given representatives

# Unlinkability of signatures (Adaption)

• Adapted signatures indistinguishable from fresh ones

Turned out to be a very versatile tool

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- Instead randomize message and adapt signature
- (Delegatable) anonymous credentials [HS14, DHS15, FHS19, CL19]
- Self-blindable certificates [BHKS18]
- Round-optimal blind signatures [FHS15, FHKS16]
- Group signatures [DS18, BHKS18, CS18, BHS19]
- Verifiably encrypted signatures [HRS15]
- Access control encryption [FGKO17]
- Scalable mix-nets [HPP19]

# Example: Simple Anonymous Credentials v2.0



# Formal Framework

#### SPS-EQ

 $\mathsf{par} \leftarrow \mathsf{ParGen}(\mathsf{1}^\lambda)$ 

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 $(\sigma, \tau) \leftarrow \mathsf{Sign}([\mathbf{m}]_i, \mathsf{sk})$ 

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par  $\leftarrow$  ParGen(1 $^{\lambda}$ )\\allow others pars beyond BG(sk, pk)  $\leftarrow$  KeyGen(par,  $\ell$ )( $\sigma, \tau$ )  $\leftarrow$  Sign([m]<sub>i</sub>, sk)\\allow tag  $\tau$ ([m']<sub>i</sub>,  $\sigma'$ )  $\leftarrow$  ChgRep([m]<sub>i</sub>, ( $\sigma, \tau$ ),  $\mu$ , pk)

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#### SPS-EQ

$par \gets ParGen(1^\lambda)$	\\allow others pars beyond <b>BG</b>
$(sk,pk) \leftarrow KeyGen(par,\ell)$	
$(\sigma,  au) \leftarrow Sign([\mathbf{m}]_i, sk)$	\\allow tag $ au$
$([\mathbf{m}']_i, \sigma') \leftarrow ChgRep([\mathbf{m}]_i, (\sigma, \tau),$	$\mu$ , pk)
$\{0,1\} \leftarrow Verify([\mathbf{m}]_i,(\sigma,\tau),pk)$	\\w/o tag $ au$
${ extsf{0,1}} \leftarrow { extsf{VKey}( extsf{sk,pk})}$	

Tag-based schemes have one-time randomizability

• Only  $(\sigma, \tau)$  from Sign can be put into ChgRep (enough for almost all applications)

#### EUF-CMA Security



 $\begin{array}{l} \mathsf{par} \leftarrow \mathsf{ParGen}(1^{\lambda}) \\ (\mathsf{sk},\mathsf{pk}) \leftarrow \mathsf{KeyGen}(\mathsf{par},\ell) \end{array}$ 



 $\begin{array}{l} \text{Win}:\\ \text{Verify}([\mathbf{m}]_1^*,\sigma^*,\mathsf{pk})=1 \quad \wedge\\ [\mathbf{m}]_{\mathcal{R}}^*\neq [\mathbf{m}]_{\mathcal{R}} \end{array}$ 

#### Weak EUF-CMA Security [FG18]



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Assume DDH in  $\mathbb{G}_1^*$   $\checkmark$ 

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Unlinkability of Signatures (Adaption)

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Keys and/or signature generated honestly or maliciously? Turns out to be quite subtle for applications

	Keys	Signatures
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In addition: Honest parameter model (HP)

- **par** generated honestly, but keys can be generated maliciously
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- $\cdot$  (MK,MS) in HP gives (HK,MS)

- $\cdot$  (HK,HS) introduced in [FG18]
- $\cdot\,$  (HK,MS) and (MK,MS) introduced in [FHS15]

# Overview of the State-of-the-Art

Scheme	Unforgeability	Assumption	Adaption
[FHS15]	EUF-CMA	GGM	MK, MS

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\*\*Adaption under honest keys and signatures (HK, HS) too weak for most applications (see Paper for details)

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# EUF-CMA Secure SPS-EQ from Standard Assumptions

Common technique to construct (tightly secure) SPS under standard assumptions

 $\cdot$  One-time (SP) MAC  $\longmapsto$  Many-time (SP) MAC  $\longmapsto$  SPS

Numerous works [BKP14,KW15,KPW15,GHK17,GHKP18,AJOPRW19]

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Numerous works [BKP14,KW15,KPW15,GHK17,GHKP18,AJOPRW19]

- Weakly EUF-CMA secure SPS-EQ in [FG18] use [BKP14] as starting point
- We use [GHKP18] as a starting point

### Starting from the MAC of [GHKP18]



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Hurdles to overcome

- Make MAC linear to switch within class
- $\cdot$  Have malleable and perfectly randomizable proofs  $\Omega$

# Starting from the MAC of [GHKP18]



Hurdles to overcome

- Make MAC linear  $\checkmark$  to switch within class
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# Doubling of a modified MAC of [GHKP18]



First steps

- Add second "MAC" (to empty message), which acts as tag
- \* Doubling OR-NIZK, sharing randomness
- Fix k = 1 (**A**<sub>0</sub>, **A**<sub>1</sub> vectors) instantiation from SXDH only

# Achieving Malleability and Perfect Randomizability

Modify the OR-NIZK of [GHKP18]

Problem

• [GHKP18] fixes  $[\mathbf{Z}]_2$  in CRS and provide  $[\mathbf{Z}_0]_2$  and  $[\mathbf{Z}_1]_2$  s.t.  $[\mathbf{Z}]_2 = [\mathbf{Z}_0]_2 + [\mathbf{Z}_1]_2$  and at least one is in span( $\mathbf{Z}$ )

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Replace this part with a homomorphic QA-NIZK [JR14]

- Show that one of  $[\mathbf{z}_0]_2$  and  $[\mathbf{z}_1]_2$  is in span $(\mathbf{D} + \mathbf{z})$
- Preservers the soundness of OR-NIZK

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#### Malleable $\checkmark$ and perfectly randomizable $\checkmark$ proofs

Now supports additive update of the two OR-NIZK yielding a perfectly distributed fresh proof for witness  $r' = r_1 + \psi r_2$  and word  $[t']_1 = \mu[t]_1 + \psi[w]_1$ 

Scheme	Signature	PK	Ass.	Red. Loss
[FHS15]	$2 \mathbb{G}_1 +1 \mathbb{G}_2 $	$\ell \mathbb{G}_2 $	GGM	-
[FG18]	$(4\ell+2) \mathbb{G}_1 +4 \mathbb{G}_2 $	$(4\ell+2) \mathbb{G}_2 $	DLIN	$\mathcal{O}(Q)$
This work	$8 \mathbb{G}_1 +9 \mathbb{G}_2 $	$3\ell \mathbb{G}_2 $	SXDH	$\mathcal{O}(\log Q)^*$

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\*Tightness inherited from [GHKP18]

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- Access control encryption (ACE) in [FGKO17]
- Self-blindable certificates [BHKS18]
- Attribute-based credentials wo malicious issuer (or with a CRS) [HS14, FHS19]
- Shortest round-optimal blind signatures with a CRS; improving by about a factor of 4 (using the template in [FHS15,FHKS16])

Take Home & Open Questions

# Conclusion

#### Take Home

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#### **Open Questions**

- Apply our idea to other SPS to improve efficiency and/or support other assumptions
- Construct SPS-EQ under standard assumption that support malicious keys wo HP, i.e., (MK,MS)
  - · Constructions wo CRS seem very hard
  - Untrusted CRS?

# Thank you! Questions?

♥ @drl3c7er

Supported by EU ECSEL



and FWF/netidee SCIENCE PROFET

Der Wissenschaftsfonds.

