Modern Cryptography

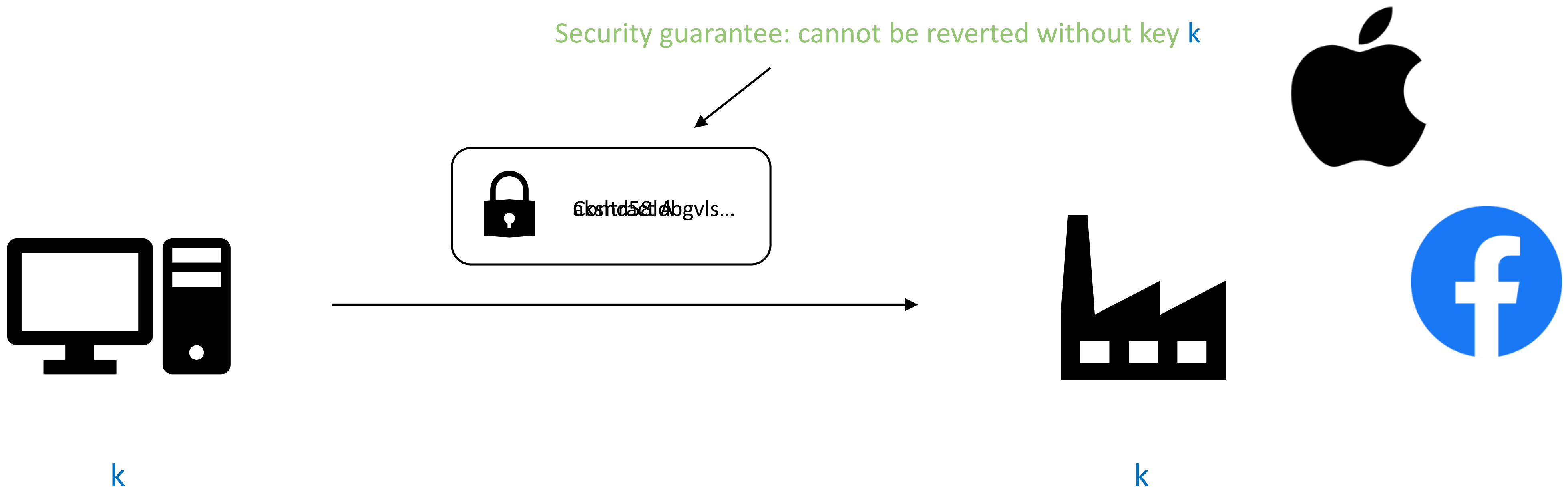
Lecture 15, Advanced (Public-Key) Encryption

Christoph Striecks

Organizational

- Where to find the slides and homework?
  - https://danielslamanig.info/ModernCrypto19.html
- How to contact us?
  - {Daniel.Slamanig, Christoph.Striecks}@ait.ac.at
- Tutor: Guillermo Perez, Karen Klein
  - guillermo.pascualperez@ist.ac.at, karen.klein@ist.ac.at
- Official page at TU, Location etc.
  - https://tiss.tuwien.ac.at/course/courseDetails.xhtml?dswid=3463&dsrid=417&courseNr=192062&semester=2019W
- Tutorial, TU site
  - https://tiss.tuwien.ac.at/course/courseDetails.xhtml?dswid=3593&dsrid=246&courseNr=192063
- Exam for the second part: Thursday 30/01/2019 15:00-17:00 (Tutorial slot)
  - Both exams will be taken into considerations concerning grading

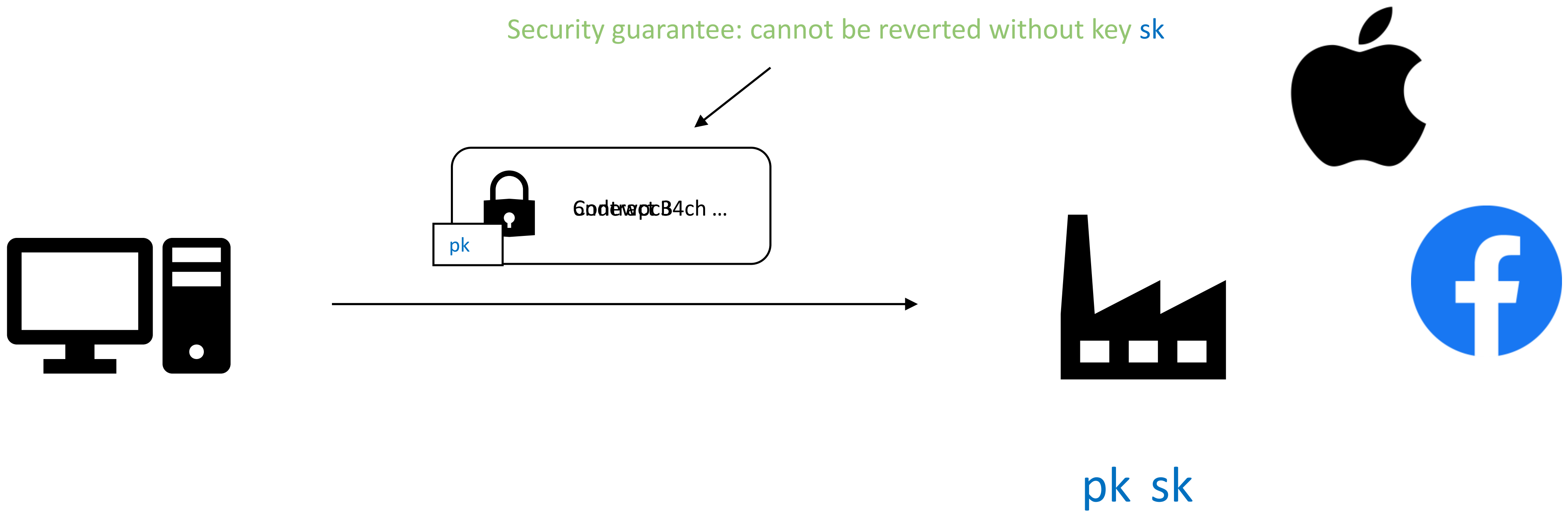
Crypto 1.0: Secret-Key Encryption



✓ Enables secure one-to-one communication

× Key k has to be distributed before encryption (problem: how?)
× Keys have to be securely maintained for each communication partner (i.e., huge overhead)
× Encryption is all-or-nothing (keys have one-to-one relationship)

Crypto 2.0: Public-Key Encryption



- ✓ Enables secure one-to-one communication
- ✓ Solves key-distribution problem (pk can be publicly available) compared to secret-key encryption

- × Public key pk has to be authenticated, e.g., by using heavy **Public-Key Infrastructures (PKI)**
- × Encryption is all-or-nothing

Recall Public-Key Encryption

- Gen(1^n): on input security parameter 1^n, return public and secret keys (pk, sk), where message space *M* is defined in pk.

- Enc(pk,m): on input pk and message m, return ciphertext c

- Dec(sk,c): on input sk and c, return m or error

- Correctness: for all integer k, for all (pk,sk) ← Gen(1^n), for all messages m, for all c ← Enc(pk,m), we have that m = Dec(sk,c) holds except with negl. probability.

- Security: OW-CPA, IND-CPA, IND-CCA notions

Identity-Based Encryption

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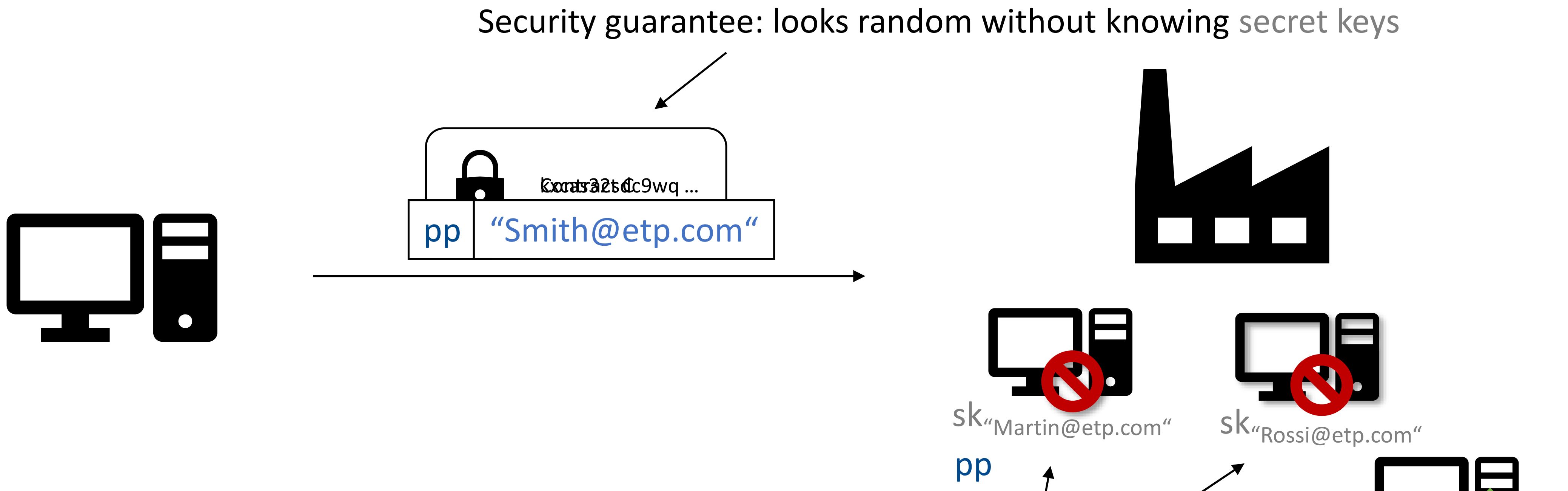
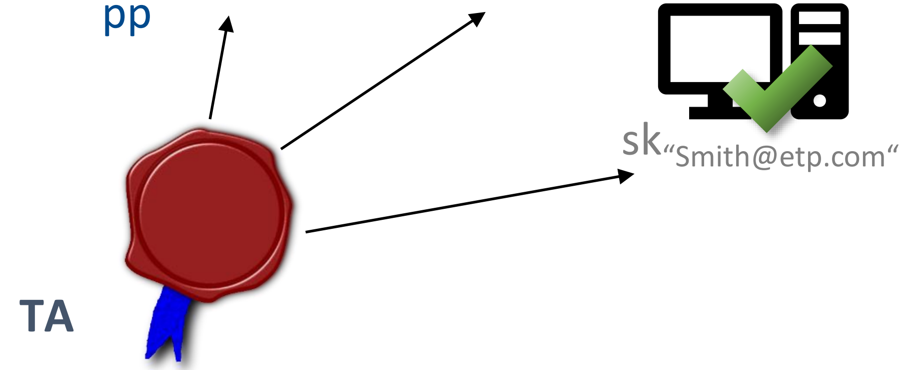
Crypto 3.0: Identity-Based Encryption



Properties:

- Public key is a public string, e.g., email address
  - Essentially compress exponentially many pk’s into short pp
- Many user secret keys for one pp („less-heavy“ PKI needed)
- Need of pp-related authority **TA** that distributes keys



Identity-Based Encryption, Definition*

DEFINITION. An IBE scheme Ξ with **identity** and message spaces ***ID*** and *M*, respectively, consist of four PPT algorithms (Gen, **Ext**, Enc, Dec) such that:

- Gen(1^k): on input security parameter 1^k, return public parameters and secret key (pp, sk), where message space *M* **and identity space *ID*** is defined in pp.
- **Ext(sk,id): on input identity id and secret key sk, return user secret key usk_{id}.**
- Enc(pp,m,**id**): on input public parameter pp, **identity id ϵ *ID***, and message m ϵ *M*, return ciphertext $\mathbf{c_{id}}$
- Dec($\mathbf{usk_{id}}$,$\mathbf{c_{id}}$): on input secret key $\mathbf{usk_{id}}$ and ciphertext $\mathbf{c_{id}}$, return m or error
- **Correctness**: for all integer k, for all (pp,sk)←Gen(1^k), **for all identities id ϵ *ID***, **for all usk_{id}**← **Ext(sk,id)**, for all messages m **ϵ** *M*, for all $\mathbf{c_{id}}$←Enc(pp,**id**,m), we have that m=Dec($\mathbf{usk_{id}}$,$\mathbf{c_{id}}$) holds except with negl. probability.
- **Security**: IBE-IND-CPA and IBE-IND-CCA notions (plus variants thereof)

*We highlight the main differences to PKE with **bold**.

Some Remarks on the IBE Definition

- As in PKE, encryption may be deterministic or probabilistic
- As in PKE, decryption may be perfectly correct or may fail with negl. probability

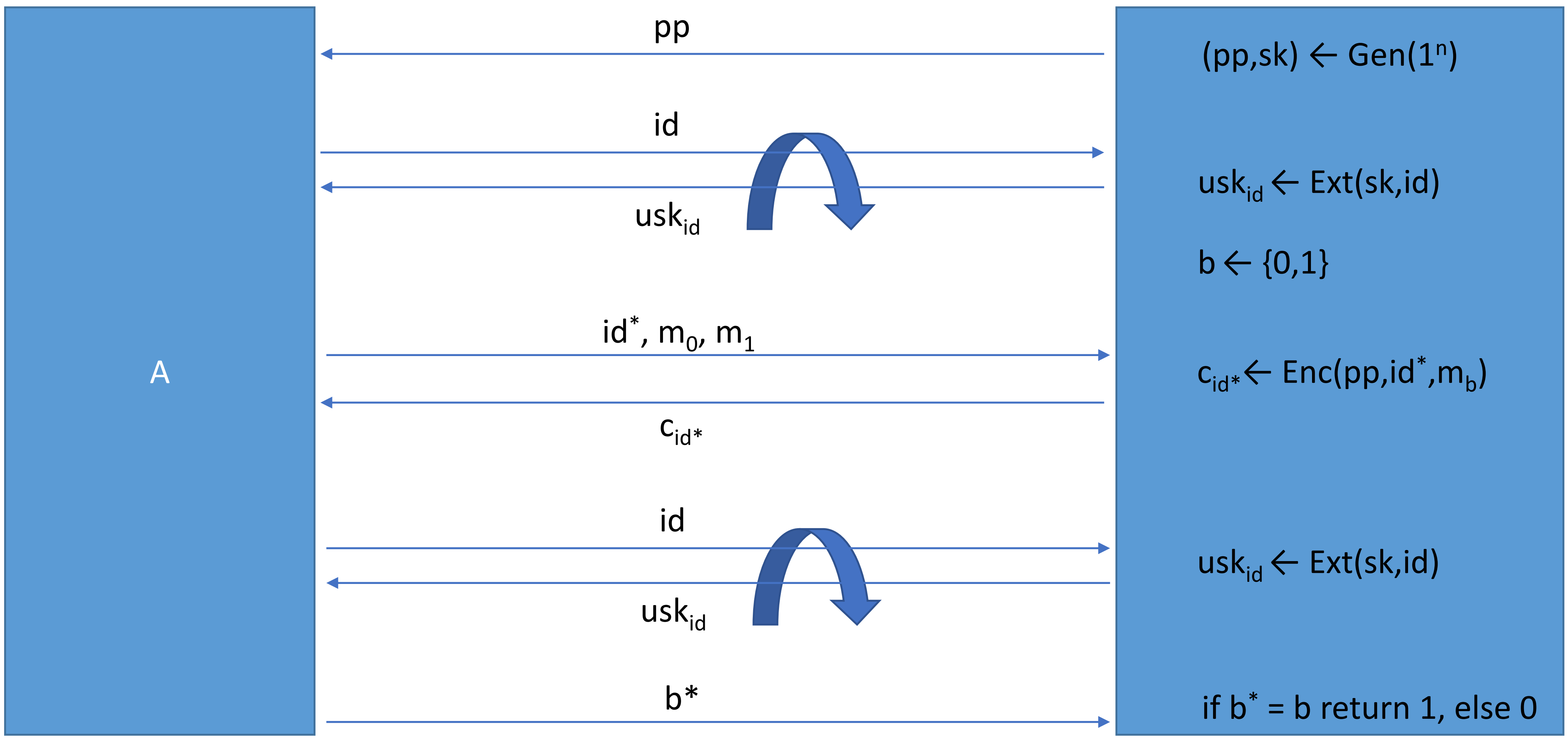
- Opposed to PKE, an identity space is defined which is typically exponentially large (question: why?)
  - This also means exponentially many user secret keys possible and, hence, constitute a multi-user encryption system

  - But: trusted authority is needed to generate user secret keys

Security Definitions (Initial Thoughts)

- IBE scheme is a multi-user system
  - **Multiple user secret keys** can be compromised (**not the case in PKE IND-CPA security**)
  - Attacker should be able to retrieve user-secret keys of its choice (**not the case in PKE IND-CPA security**)
  - Similarly to PKE IND-CPA, attacker should not be able to distinguish ciphertexts of chosen messages and "target identity" (question: what must be realized by a security definition to exclude trivial wins?)
  - We will dub the security notions for IBE as IBE-IND-CPA

IBE-IND-CPA Security: $\text{Exp}_{IBE,A}$



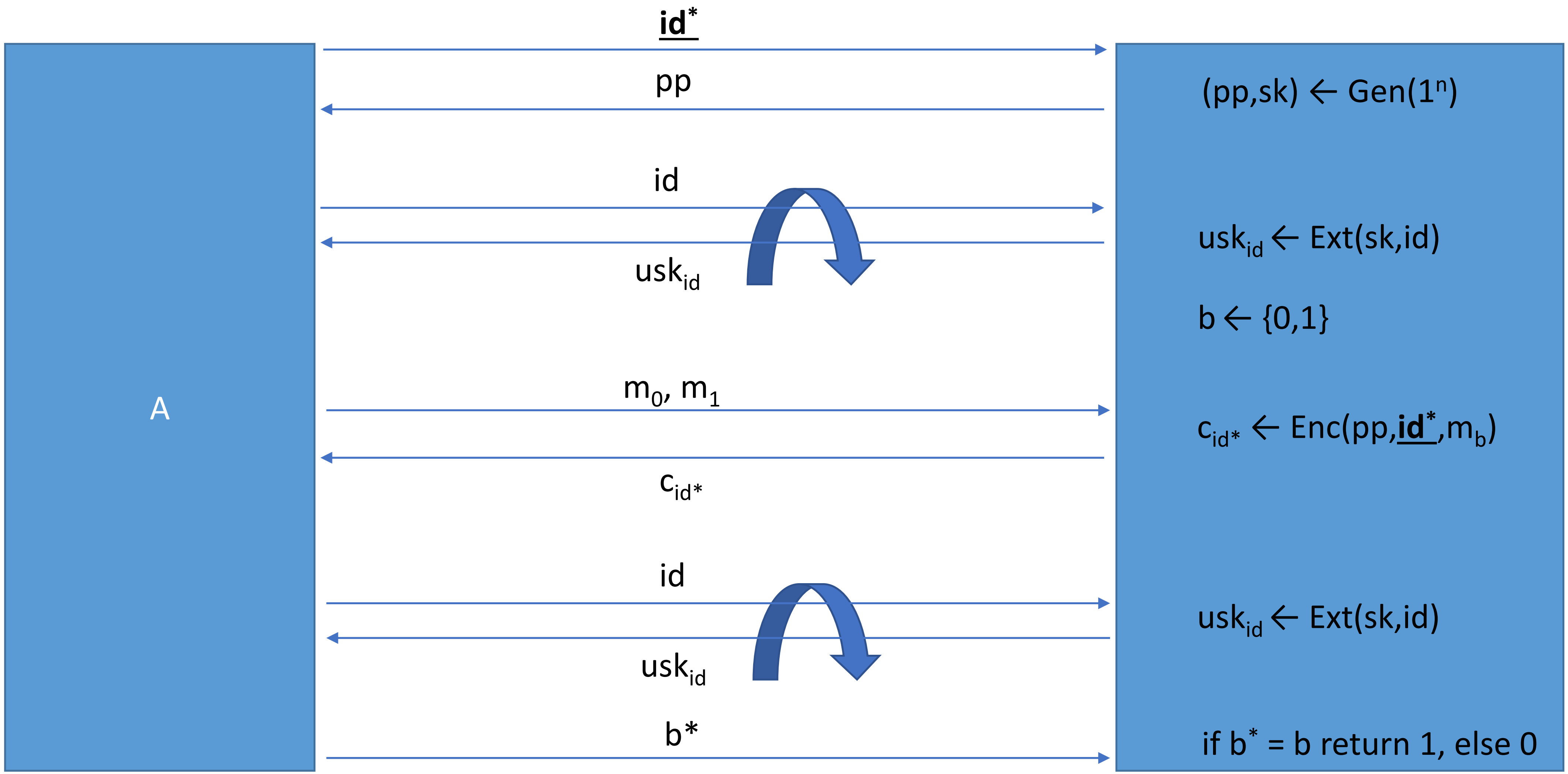
IBE-IND-CPA Security

Definition. An IBE scheme Ξ = (Gen, Ext, Enc, Dec) is IBE-IND-CPA secure if and only if $Adv_{IBE,A}(1^n) := |Pr[Exp_{IBE,A}(1^n)=1] - ½|$ is negl. in n, for any valid PPT adversary *A* and $|m_0|=|m_1|$. *A* is valid if id^* was never queried by *A*.

- Remark: IBE-IND-CPA security is very hard to achieve

- That is the reason why the first schemes were only proven secure in a weaker model dubbed Weak-IBE-IND-CPA

Weak-IBE-IND-CPA Security: $Exp_{Weak\text{-}IBE,A}$



Weak-IBE-IND-CPA Security

Definition. An IBE scheme Ξ = (Gen, Ext, Enc, Dec) is Weak-IBE-IND-CPA secure if and only if $\mathrm{Adv}_{\text{Weak-IBE},A}(1^n) := |\Pr[\mathrm{Exp}_{\text{Weak-IBE},A}(1^n)=1] - ½|$ is negl. in n, for any valid PPT adversary *A* and $|m_0|=|m_1|$. *A* is valid if id^* was never queried by *A*.

- Indeed, many system in the literature were constructed to be "only" Weak-IBE-IND-CPA secure
  - IBE-IND-CPA in Standard Model (without RO) hard to achieve (only 2005 with large parameters by Waters)
  - However, *inefficient* generic transformations (from Weak-IBE-IND-CPA to IBE-IND-CPA) are known

Constructing IBE

- Interestingly, constructing IBE is harder compared to constructing PKE
  - Often: mathematical “trick” necessary, i.e., **pairing**
  - Up to now, only a few (inefficient) schemes exist that do not rely on pairings (e.g., best paper from CRYPTO 2017 under DDH, or Cocks’ scheme from factoring)

- Proposed by Shamir in the 1984, first realizations only 2001 due to Boneh and Franklin, and Cocks

- IBE is building block for: digital signatures, searchable encryption, IND-CCA secure PKE, forward-secret encryption (puncturable encryption)

Strong Tool: Pairings

- Given cyclic groups G, G_T with prime-order p
- Furthermore, given a mapping e: G x G -> G_T and generator g ϵ G

- Properties
  - Non-degeneracy: for all g ϵ G, g ≠ 1, e(g,g) ≠ 1 holds.
  - Bilinearity: for all g ϵ G and integers a,b, $e(g^a,g^b) = e(g^b,g^a)$ holds. (In part., e(g,h)=e(h,g), why?)
- DDH assumption might not hold in G, since one can efficiently test DDH tuples (as a result, Bilinear DH assumption was introduced, also used in IBE constructions and further)

Boneh-Franklin (BF) IBE

- Assume (e, G, G_T, p, g) and Random Oracle $H : ID \to G$, message and identity spaces *M* and *ID*, resp., are given as input to each algorithm
- BF.Gen(1^k): return (pp, sk) := (g^x, x), for g in G
- BF.Ext(id,sk): return $usk_{id} := H(id)^x$
- BF.Enc(pp,id,m): return $c_{id} := (c_1, c_2) := (g^y, e(g^x, H(id))^y * m)$
- BF.Dec(usk_{id},c_{id}): return $c_2 / e(c_1, usk_{id})$
- Correctness holds (why?):
  - $e(c_1, usk_{id}) = e(g,H(id))^{xy}$ and $e(g,H(id))^{xy} * m = c_2$

Bilinear Diffie-Hellman Assumption

- Bilinear DH (BDH) assumption is an extension of the computational DH assumption to the pairing setting
  - Essentially: given g^x, g^y, g^z it is hard to compute $e(g,g)^{xyz}$
- Security of BF IBE: IBE-IND-CPA secure in the RO model under BDH assumption
- Many schemes in the Standard-Model were only proven Weak-IBE-IND-CPA secure until 2009 (Waters)
- Nowadays: many IBE-IND-CPA and IBE-IND-CCA schemes are known and constitute state-of-the-art

“Homework”: Naor’s Tranformation

- Interesting observation: each IBE scheme is also a signature scheme due to Naor (described in Boneh-Franlin IBE paper from 2001)

- Sketch:
  - Signature public and secret keys (pp, sk) are public parameter and secret key of the IBE.Gen, respectively
  - The signature σ is the output of IBE.Ext with “identity” m and sk (where m is the message in the signature scheme)
  - Verification of a signature σ and a message m is done by running IBE.Enc with pp and random message R and m; and try decrypting the resulting ciphertext with the signature σ
  - If the result of the decryption yields R, then the signature is valid for m under pp
  - Correctness? Homework …

Attribute-Based Encryption

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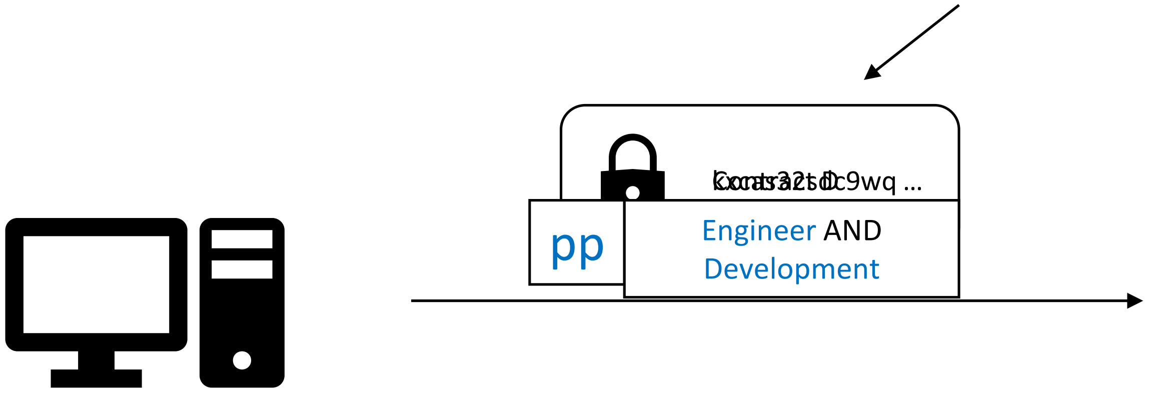
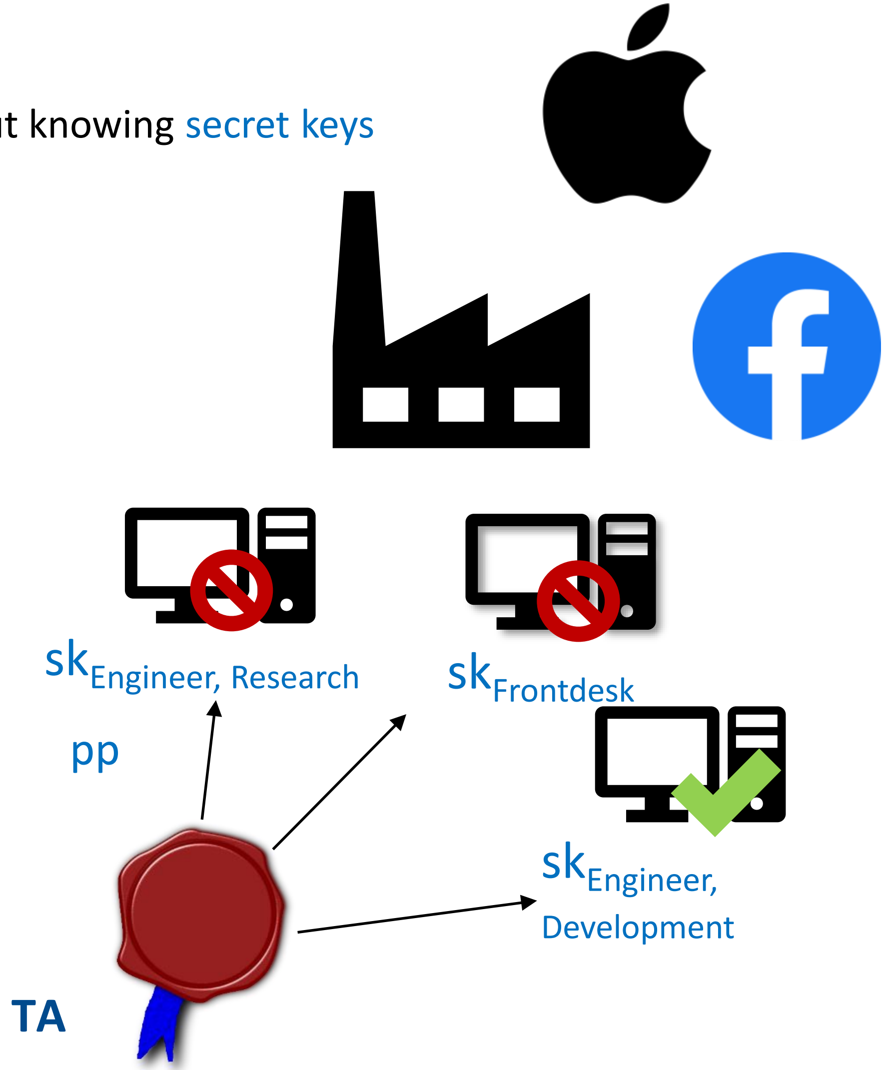
AUSTRIAN INSTITUTE OF TECHNOLOGY
TOMORROW TODAY

Crypto 4.0: Attribute-Based Encryption

Security guarantee: looks random without knowing secret keys





- ✓ Enables **fine-grained** one-to-many communication
- ✓ Many secret keys (hence, less-heavy PKI needed)
- ✓ **Bonus feature: enforces access control on the cryptographic level**

× Central authority **TA** that controls and distributes secret keys

Attribute Based Access Control

https://csrc.nist.gov/projects/abac/

NIST

Information Technology Laboratory

COMPUTER SECURITY RESOURCE CENTER

PROJECTS

Attribute Based Access Control

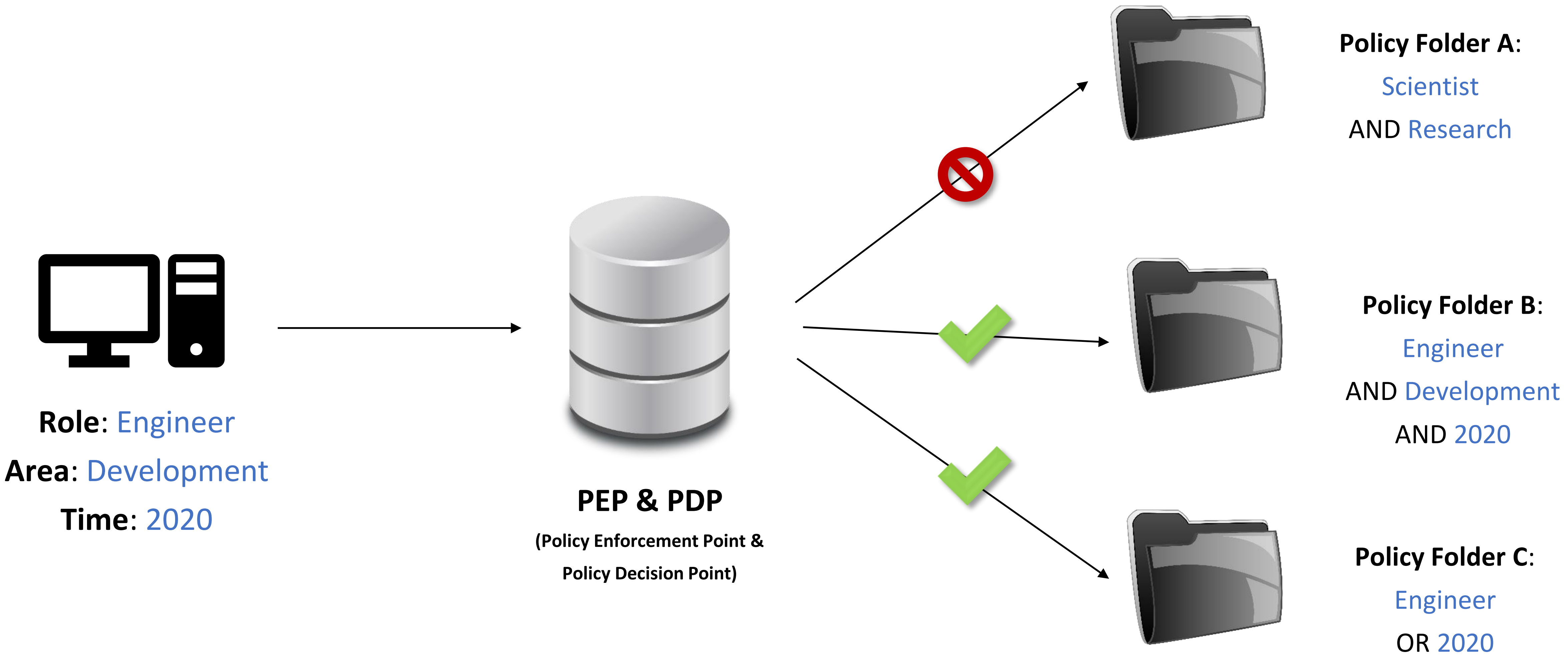
Project Overview

The co | um of
logical | ible
metho

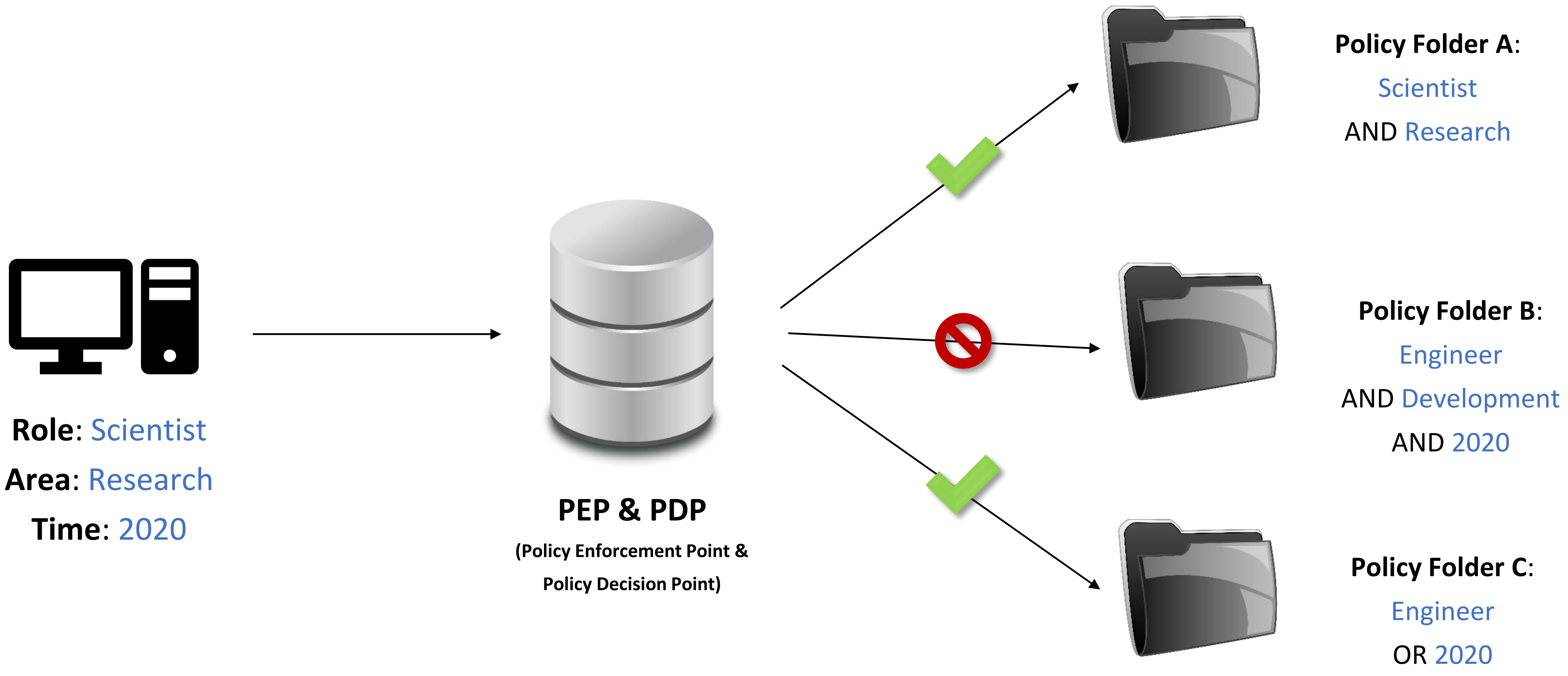
NIST Special Publication 800-162 (Jan. 2014)

In Nove | ity,
Creden | federal
organiz | enable
access within and between organizations across the Federal enterprise. In December 2011, the FICAM Roadmap and Implementation Plan v2.0 took the next step of calling out ABAC as a recommended access control model for promoting information sharing between diverse and disparate organizations.

Attribute-Based Access Control (ABAC, simplified)



Attribute-Based Access Control (ABAC, simplified)



Attribute-Based Access Control (ABAC)

- Advantage: fine-grained access to data, defined on attributes and policies with strong PEP/PDP mechanisms
- Disadvantage: massive trust in software-based PEP/PDP implementations (software implementation often prone to errors)

Can we do better?

Yes! Enforcing access control through cryptography using Attribute-Based Encryption (ABE)

Initial Thoughts on ABE

- Attributes and Policies play essential part in ABE
  - Attributes are (bit) strings
  - Policies can be seen as Boolean formulas, e.g., („Scientist“ AND „Research“) OR „Engineer“
  - Informal for now: we say „an attribute set satifies a policy“ if the Boolean formula evaluates to true for an attribute input set

- Where to put attributes? Ciphertext, Keys?
- Where to put policies? Ciphertext, Keys?

- Hence, two variants of ABE exist
  - **Key-Policy ABE (KP-ABE):** ciphertexts hold attributes, keys hold policies

  - **Ciphertext-Policy ABE (CP-ABE):** ciphertexts hold policies, keys hold attributes

KP-Attribute-Based Encryption, Definition

DEFINITION. A KP-ABE scheme Ω_{KP} consist of four PPT algorithms (Gen, **Ext**, Enc, Dec) such that:

- Gen(1^k): on input security parameter 1^k, return public parameters and secret key (pp, sk), where message space *M* **and attribute space *A*** and **policy space *P*** is defined in pp.
- **Ext(sk,p): on input secret key and policy p ϵ *P*, return user secret key usk_p.**
- Enc(pp,**a**,m): on input public parameter pp, **attribute set a ϵ *A***, and message m ϵ *M*, return ciphertext c_a
- Dec(usk_p,c_a): on input secret key usk_p and ciphertext c_a, return m **if a satisfies p**, or error
- Correctness: for all integer k, for all (pp,sk) ← Gen(1^k), **for all attribute sets a ⊂ *A***, **for all policies p ϵ *P***, **for all usk_p** ← **Ext(sk,p)**, for all messages m, for all c_a ← Enc(**a**,**pp**,m), we have that m=Dec(usk_p ,c_a) holds **if a satisfies p** except with negl. probability.
- Security: KP-ABE-IND-CPA (on slide 28), KP-ABE-IND-CCA notions

CP-Attribute-Based Encryption, Definition

DEFINITION. A CP-ABE scheme Ω_{KP} consist of four PPT algorithms (Gen, **Ext**, Enc, Dec) such that:

- Gen(1^k): on input security parameter 1^k, return public parameters and secret key (pp, sk), where message space *M* **and attribute space *A*** and **policy space *P*** is defined in pp.
- **Ext(sk,a): on input secret key and attribute set a ϵ *A*, return user secret key usk_a.**
- Enc(pp,**p**,m): on input public parameter pp, **policy p ϵ *P***, and message m ϵ *M*, return ciphertext $\mathbf{c_p}$
- Dec($\mathbf{usk_a}$,$\mathbf{c_p}$): on input secret key $\mathbf{usk_a}$ and ciphertext $\mathbf{c_p}$, return m **if a satisfies p**, or error
- Correctness: for all integer k, for all (pp,sk) ← Gen(1^k), **for all attribute sets a ⊂ *A***, **for all policies p ϵ *P***, **for all usk_a** ← **Ext(sk,a)**, for all messages m, for all $\mathbf{c_p}$ ← Enc(pp,**p**,m), we have that m = Dec($\mathbf{usk_a}$,$\mathbf{c_p}$) holds **if a satisfies p** except with negl. probability.
- Security: CP-ABE-IND-CPA (on slide 30), CP-ABE-IND-CCA notions

Some Remarks on the Definition

- As in IBE, encryption may be deterministic or probabilistic
- As in IBE, decryption may be perfectly correct or may fail with negl. probability
- As in IBE, exponentially many user secret keys possible and, hence, constitute a multi-user encryption system

- Opposed to IBE, an attributes space **and** a policy space is defined
- As in IBE, trusted authority is needed to generate user secret keys

Security Definitions (Initial Thoughts)

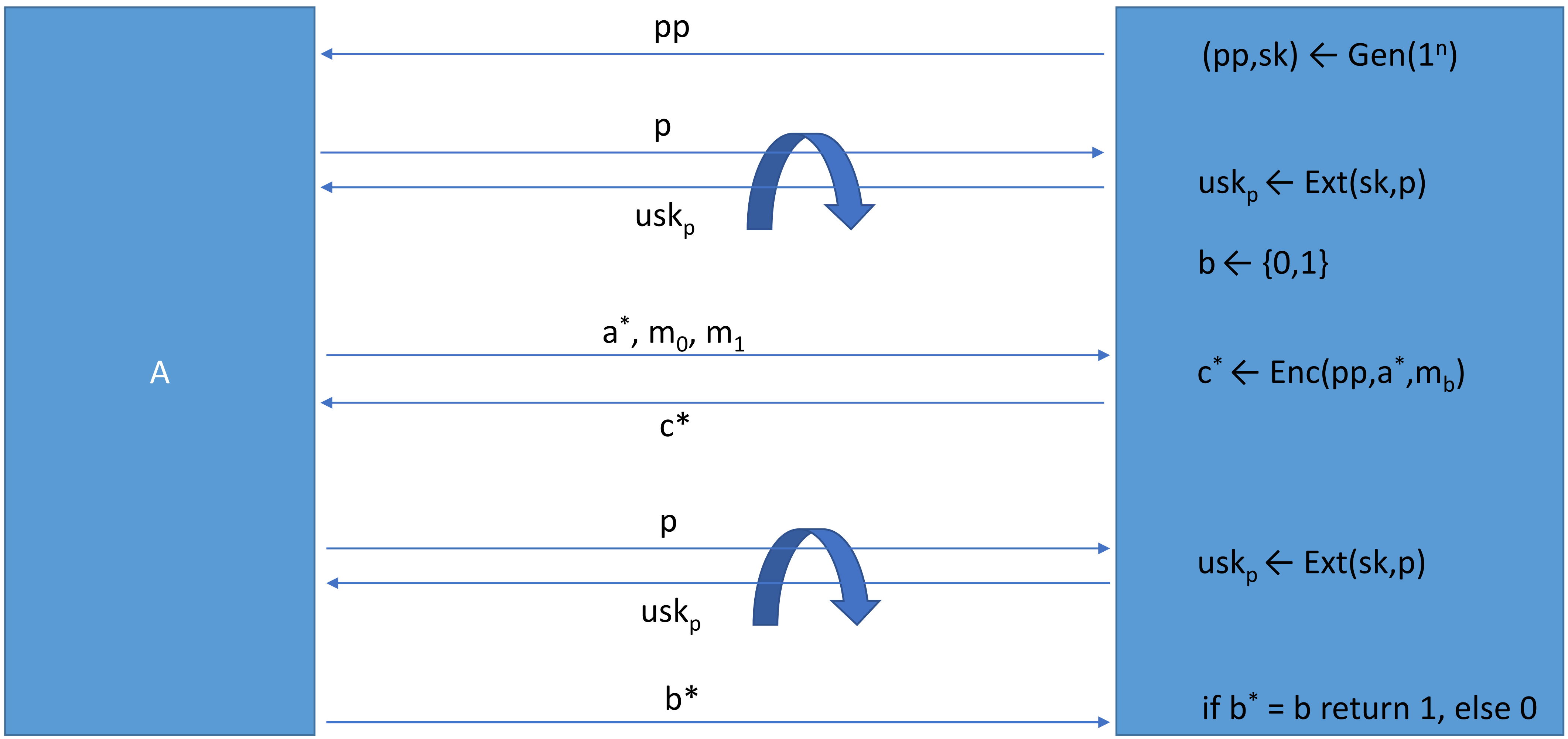
- ABE scheme is a multi-user system
  - Multiple user secret keys can be compromised (and combined)
  - Distinguishing feature in ABE: **collusion resistance of private keys**!

  - Attacker should be able to retrieve user secret keys of its choice

  - Similarly to IBE-IND-CPA, attacker should not be able to distinguish ciphertexts of chosen messages and “attribute set” of “policy” (question: what must be realized by a security definition to exclude trivial wins?)

  - We will dub the security notions for KP-ABE and CP-ABE as KP-ABE-IND-CPA and CP-ABE-IND-CPA, respectively

KP-ABE-IND-CPA Security: $Exp_{KP\text{-}ABE,A}$

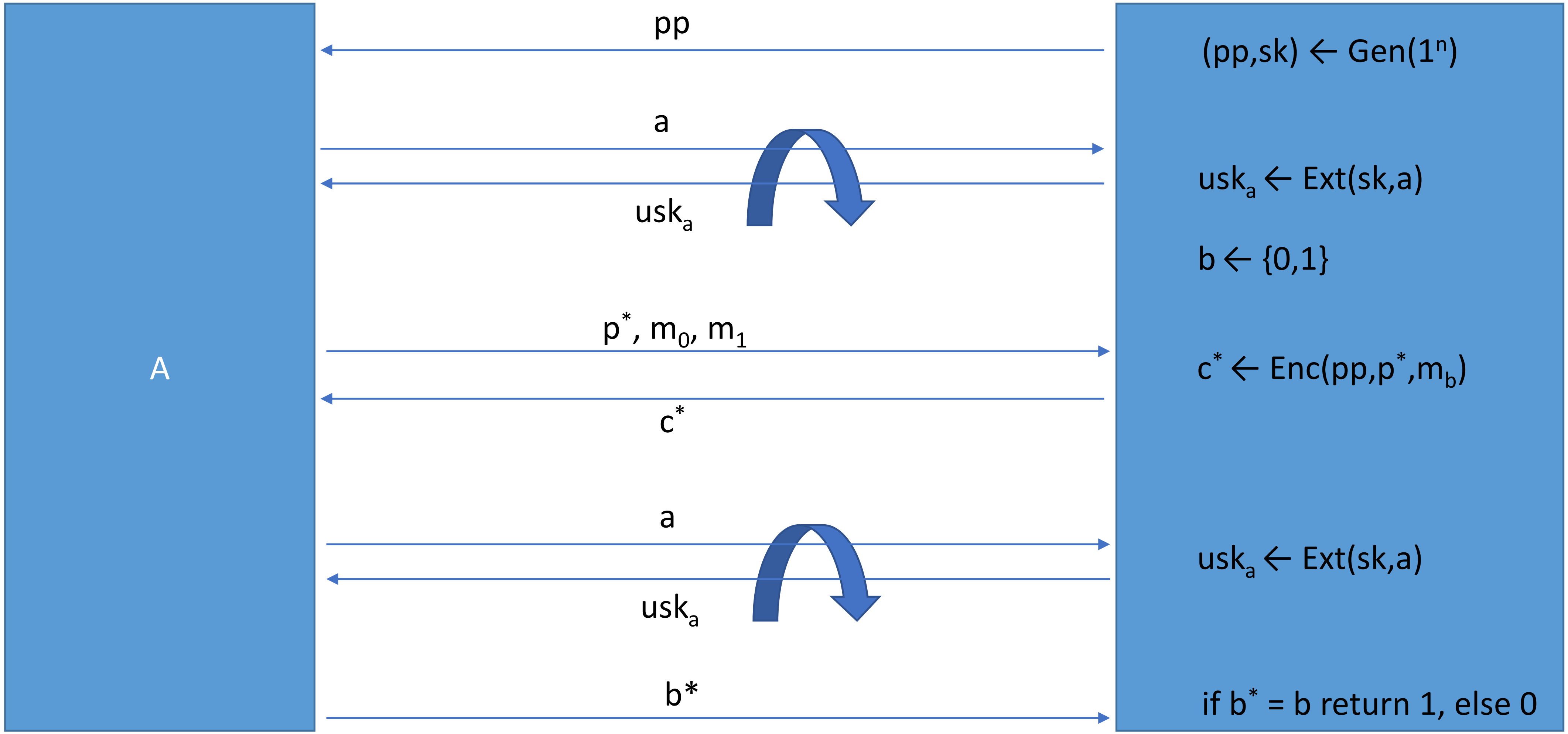


KP-ABE-IND-CPA Security

Definition. An KP-ABE scheme Ω = (Gen, Ext, Enc, Dec) is KP-ABE-IND-CPA secure if and only if $Adv_{KP\text{-}ABE,A}(1^n) := |Pr[Exp_{KP\text{-}ABE,A}(1^n)=1] - ½|$ is negl. in n, for any valid PPT adversary A and $|m_0|=|m_1|$. A is valid if a^* does not satisfy any queried policy.

- Remark: KP-ABE-IND-CPA security is very hard to achieve indeed
- Similar to IBE, the first ABE schemes were only proven secure in a weaker model

CP-ABE-IND-CPA Security: $Exp_{CP\text{-}ABE,A}$



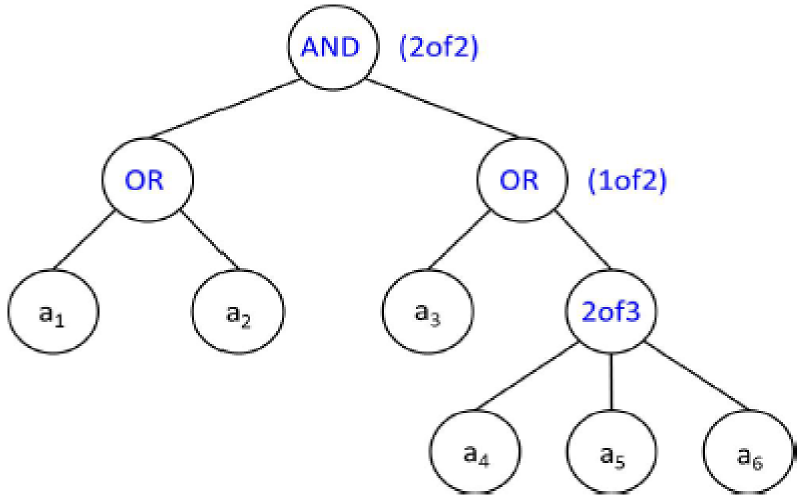
CP-ABE-IND-CPA Security

Definition. An CP-ABE scheme Ω = (Gen, Ext, Enc, Dec) is CP-ABE-IND-CPA secure if and only if $Adv_{CP\text{-}ABE,A}(1^n) := |Pr[Exp_{KP\text{-}ABE,A}(1^n)=1] - ½|$ is negl. in n, for any valid PPT adversary A and $|m_0|=|m_1|$. A is valid if any queried a does not satisfy p^*.

- Remark: CP-ABE-IND-CPA security is very hard to achieve as well

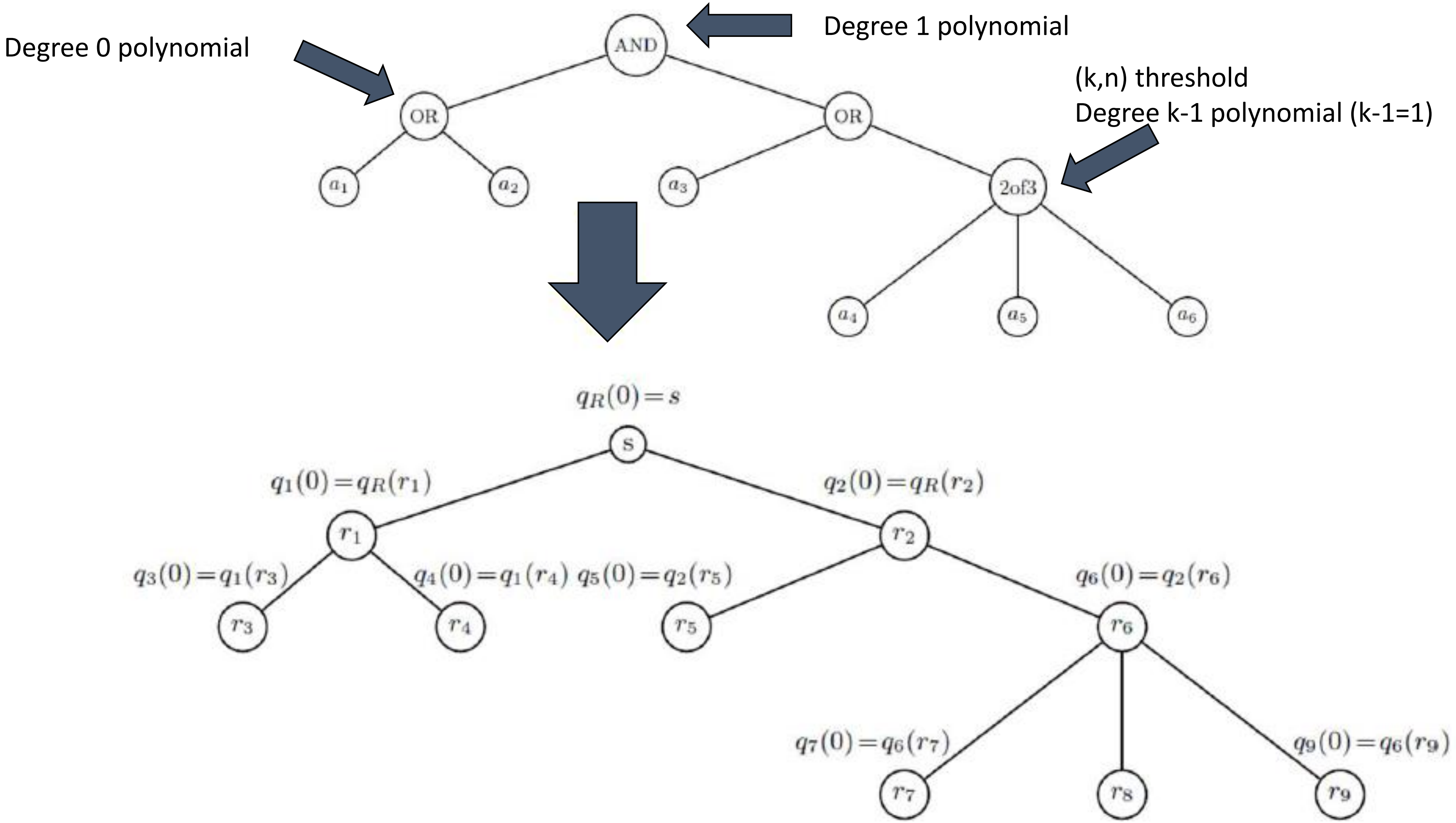
Constructing CP-ABE (Bethencourt, Sahai, Waters, IEEE S&P 2007)

- The policy is put into the ciphertext and user secret keys are issued for sets of attributes
- Main ingredients: „access trees“, pairings, and secret sharing
  - Let A = $\{a_1,\ldots,a_6\}$ be the set of attributes, with policy p=(a_1 **OR** a_2) **AND** (a_3 **OR** **2of3**(a_4,a_5,a_6))*:



* Here, we also allow a threshold gate **2of3**.

CP-ABE Idea (BSW07)



CP-ABE Idea (BSW07)

- Public key (system parameters)

$$pk = (g, h = g^{\beta}, e(g,g)^{\alpha})$$

- User with attribute set A = {a_1,…,a_n} gets private key

$$(D = g^{(\alpha+r)/\beta}, (D_i = g^{r} \cdot H(a_i)^{r_i}, D_i' = g^{r_i})_{a_i \in A})$$

- Keys are randomized per user $(r, r_1, \ldots, r_n)$ to avoid collusion attacks
- Ciphertexts for policy (access tree) including all leafs j and with root of tree

$$q_R(0) = s$$

$$C' = m \cdot e(g,g)^{\alpha s}, C = h^{s}, (C_j = g^{q_j(0)}, C_j' = H(a_j)^{q_j(0)})$$

CP-ABE Idea (BSW07)

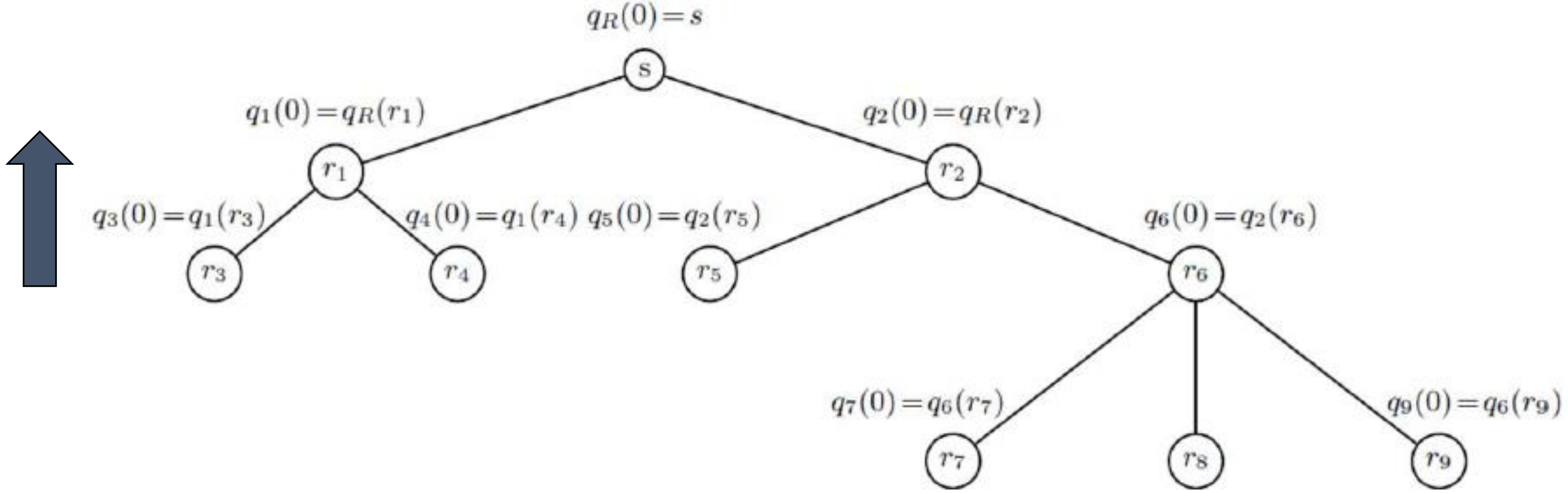
- Decryption: Start at the leaves

$$\begin{aligned}\mathsf{DecryptNode}(c, sk, j) &= \frac{e(D_j, C_j)}{e(D'_j, C'_j)} \\ &= \frac{e(g^r \cdot H(j)^{r_j}, g^{q_j(0)})}{e(g^{r_j}, H(j)^{q_j(0)})} = \frac{e(g,g)^{rq_j(0)} e(g^{r_j}, H(j)^{q_j(0)})}{e(g^{r_j}, H(j)^{q_j(0)})} \\ &= e(g,g)^{rq_j(0)}.\end{aligned}$$

- Work up the tree for all inner nodes, then remove masking
- Polynomial interpolation in the exponent
- Works if user secret key contains attributes such that the threshold of every inner node can be satisfied

CP-ABE Idea (BSW07)

$$A = e(g,g)^{r \cdot q_R(0)} = e(g,g)^{r \cdot s}$$



$$C'/(e(C,D)/A) = C'/(e(h^s, g^{(\alpha+r)/\beta})/e(g,g)^{rs}) = C'/e(g,g)^{\alpha s} = m$$